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ORIGINAL ARTICLE

When Spillovers Enhance R&D Incentives

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Abstract

It is commonly believed that spillover reduces R&D incentives of a firm. This happens because of the appropriability problem. However, some empirical literature shows the possibility of enhanced R&D incentives under spillovers. In the literature this is explained under incomplete information, but we show this theoretically under complete information. We show in particular that in a duopoly there are situations when with no spillovers only one firm invests in R&D, but under spillovers both the firms invest. This occurs when there is complementarity in research and the spillover rate lies in an interval specified by the size of R&D investment.

Keywords R&D spillovers · Appropriability problem · Complete information · R&D incentives

JEL classifications D43 · L13 · O31

Introduction

Spillovers in research and development (R&D) is a common phenomenon. It implies leakages, voluntary or involuntary, of R&D results of an innovator to its rival. Hence in the presence of R&D spillovers the innovator cannot appropriate the full benefit of its innovation. This reduces incentives for R&D investment. The existing literature clearly reveals this under-investment both theoretically and empirically. Arrow (1962)

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observed that underinvestment in R&D is likely in presence of spillovers. Similar view has been opined by Spence (1984). That a firm can benefit from R&D of its rivals is also evident in Jaffe (1986) which provides empirical evidence of the presence of spillovers. The whole literature on RJV has based its argument on this belief.[1]

However, empirically it is observed that spillovers may not always discourage investments in R&D. For instance, in an empirical study, Levin (1988) has shown that contrary to Spence's prediction, in industries like computers, communications equipment, electronic components and aircraft, spillovers have not resulted in a reduction of R&D investments.[2]

One plausible explanation offered for this observed departure of the empirical results from the theoretical prediction is that in electronics based industries, the technical advances are more of "cumulative" rather than "discrete" nature, so that innovations act like "building blocks" or foundations for innovation in the next period. Here spillover of rivals' R&D raises the marginal product of R&D of individual firms and thus the R&D investment goes up as a whole.

De Bondt (1997) finds a different reason for spillovers encouraging R&D investment by firms. Accordingly, the spillover possibilities are likely to encourage R&D as an individual firm's R&D efforts create incentives for other firms to go for similar ventures, and therefore all of them may be able to produce at lower costs resulting in lower prices and hence higher demand.

A recent work by Bakhtiari and Breunig (2018) involves an empirical study that examines the effect of spillovers on firm level R&D investment in Australia. The paper recognizes both positive and negative effects of spillovers and finds that generally negative effects dominate positive effects, but there are cases when the spillovers encourage firm level R&D investment. This is observed particularly when the firms are located within a geographical distance and engaged in complementary research.

In a more recent work, Chatterjee et al. (2018) have shown that in an incomplete information framework there are situations when the R&D investment of a firm can be larger in the presence of spillovers compared to the case of no spillovers. This is the case when the innovators have private information about their respective absorptive capacities to acquire, fully or partially, the rival's innovation. Given that the rival's spillover parameter is unknown to the concerned firm, if its absorptive capacity is not large enough, it may invest more in R&D. Then it remains a question of whether the similar result will follow under complete information. This paper seeks to portray situations when even under complete information R&D incentives under spillovers can be larger than that under no spillover situation.

In the present paper we consider the scenario of d'Aspremont and Jacquemin (1988) and their followers, but assume that the R&D investment associated with an innovation is exogenously specified. In particular, we assume that if a firm invests an amount $R > 0$, it reduces its unit cost of production by an amount Δ, and if its rival also invests R, the firm concerned enjoys an additional cost reduction by an amount $\beta\Delta$ $(0 \leq \beta \leq 1)$ due to spillovers. Thus when both firms invest R in R&D, each firm

[1] For instance, see d'Aspremont and Jacquemin (1988), Suzumura (1992), Kamien et al. (1992) and Ghosh and Ghosh (2014).

[2] A comprehensive analysis on the relation between R&D investment and R&D appropriability can be found in Levin et al. (1987).

benefits from the other's investment; as a result, each firm enjoys, effectively, a cost reduction of $(1+\beta)\Delta$. So this acts like the synergy or network effect.[3]

This synergy effect can be well understood in terms of a real life example. Suppose there are two companies each producing computers. If both of them undertake cost reducing R&D, due to which one of them is able to reduce the cost of producing motherboard and the other succeeds in reducing the cost of producing processors, then both will benefit from cross spillover effects. This is because the R&D results are complementary in nature and this reduces costs along two different channels.

From the above discussion it follows that the presence of spillovers may enhance R&D investment. Hence the purpose of the present paper is to show that, given the spillovers of R&D, even under complete information framework spillovers may enhance R&D incentives of a firm compared to the no spillover case. We show in particular that when only one firm invests in R&D under no spillover situation, then under spillover effect both the firms will invest in R&D if the spillover rate lies in an interval specified by the size of R&D investment. However, if under no spillover case, no firm invests in equilibrium, then under spillover none will invest in R&D.

The organisation of the paper is as follows. In the following Sect. "Model and results" we provide the model and results of the paper. Section "Conclusion" concludes the paper.

Model and Results

Consider two firms, call firm 1 and firm 2. They play a two-stage game. In the first stage, they simultaneously and non-cooperatively decide whether they will invest in R&D or not, and then in the second stage, they play a Cournot game. Their initial unit cost of production is $c > 0$. We now assume that if a firm invests an amount $R > 0$ in R&D, it comes up with an innovation that reduces its unit cost of production by an amount D with certainty,[4] and $0 < D < c$. The market demand for the product they produce is linear and in inverse form given by $P = \max\{0, a - Q\}$ where $a > 0$ is the demand shift parameter, $Q (= q_1 + q_2)$ is the aggregate output produced in the market at price P, and q_i is the output produced by firm i. We assume that the innovation is non-drastic (with or without spillover) so that the market structure always remains to be duopoly, i.e. $a-c > D$. Later we shall derive implications of the case where D is drastic without spillover, although it remains non-drastic with spillover.

When a firm innovates, let d be the extent of spillover that goes to the firm, hence $0 \leq d \leq D$; $d = 0$ implies no spillover and $d = D$ implies cent percent spillover. Therefore, if only one firm innovates, its unit cost of production becomes $c - D$, but

3 This effect is actually absent in Chatterjee et al. (2018), as a result in their paper under complete information spillovers unambiguously reduce R&D incentives of a firm.

4 Here the parameter R is the cost associated with an innovation, hence it includes the lab set up cost, the cost of installing machines and tools, and the expenses to recruiting scientific personnel (including R&D inputs).

Table 1 Payoff matrix under complete information without spillovers (WS)

		Firm 2	
		Y	N
Firm 1	Y	$\Pi(D) - R, \Pi(D) - R$	$\Pi(2D) - R, \Pi(-D)$
	N	$\Pi(-D), \Pi(2D) - R$	$\Pi(0), \Pi(0)$

Table 2 Payoff matrix under complete information with spillovers (SS)

		Firm 2	
		Y	N
Firm 1	Y	$\Pi(D + d) - R, \Pi(D + d) - R$	$\Pi(2D - d) - R, \Pi(2d - D)$
	N	$\Pi(2d - D), \Pi(2D - d) - R$	$\Pi(0), \Pi(0)$

its rival's unit cost of production becomes $c - d$. When both the firms innovate, each firm's unit cost becomes $c - D - d$.[5]

Consider the following notations: $A = a - c$, $q(x) = \frac{A+x}{3}$, and $\Pi(x) = [q(x)]^2$ where $q(x)$ and $\Pi(x)$ are, respectively, quantity and profit of a firm.[6] Clearly, the $\Pi(x)$ function is strictly increasing and strictly convex in x. In the first stage, each firm has two strategies—either invest in R&D (denoted by Y) or do not invest (N). Therefore, in the absence of spillovers (*WS*), the payoffs of the firms for different combinations of Y and N are given by the payoff matrix in Table 1.

Similarly, in the presence of spillovers (*SS*), the payoffs of the firms are given by the payoff matrix in Table 2.

Let us now define non-strategic R&D incentive (*NS*) of a firm to be the difference between its payoffs from doing and not doing R&D when the rival does not do any R&D. Similarly, strategic R&D incentive (*S*) of a firm is the difference between its payoffs from doing and not doing R&D when the rival does R&D. Hence,

$$NS(WS) = \Pi(2D) - \Pi(0) - R, \quad S(WS) = \Pi(D) - \Pi(-D) - R,$$
$$NS(SS) = \Pi(2D - d) - \Pi(0) - R, \quad S(SS) = \Pi(D + d) - \Pi(2d - D) - R.$$

5 Following the works of d'Aspremont and Jacquemin (1988) and Kamien et al. (1992) and others, the effective cost reduction of firm i through spillover is $\varepsilon_i = \phi(R_i) + \beta\phi(R_j)$, where $\phi(R_i)$ is the amount of cost reduction if R_i is invested by firm i; $\phi(0) = 0$ and $0 \le \beta \le 1$. In our case, $R_i = R_j = R, \phi(R) = D$ and $\beta D = d$. Alternatively, we can assume that production of the final good requires one unit of each of the two inputs, say X and Y, and initial unit costs of X and Y are c_X and c_Y respectively so that its initial unit cost of production is $c = c_X + c_Y$. Now assume that firm 1 can reduce unit cost of X by D amount if it invests R in R&D. similarly, firm 2 can reduce its unit cost of Y by the same amount by investing R. By this, there is spillover of knowledge, d, from one firm to the other; $0 \le d \le D$.

6 Here "x" stands for the notation of the remaining terms other than $(a - c)$ in the numerator in the quantity and profit expressions. So x is a variable. In our paper, $\Pi(\mathrm{x}) = (\mathrm{q(x)})^2 = (\frac{a-c+x}{3})^2$. Suppose firm 1 has unit cost of production $(c - D)$ and firm 2 has unit cost $(c - d)$. Then firm 1's payoff under Cournot competition is $\Pi_1 == (\frac{a-c+2D-d}{3})^2 = \Pi(2\mathrm{D} - \mathrm{d})$ and firm 2's payoff is $\Pi_2 = \Pi(2\mathrm{d} - \mathrm{D})$, and so on.

Then,

$$S(WS) - NS(WS) = (\Pi(D) - \Pi(-D)) - (\Pi(2D) - \Pi(0)).$$

Since the $\Pi(\cdot)$ function is strictly convex and increasing, we must have,

$$S(WS) - NS(WS) < 0, \tag{1}$$

i.e., under no spillover, non-strategic R&D incentive is strictly larger than strategic R&D incentive.

To compare the strategic and non-strategic incentives under spillover, consider,

$$S(SS) - NS(SS) = \Pi(D + d) - \Pi(2d - D) - \Pi(2D - d) + \Pi(0).$$

It is easy to see that $[S(SS) - NS(SS)]$ is strictly increasing function of d with $[S(SS)-NS(SS)]_{d=0} < 0, [S(SS)-NS(SS)]_{d=D} > 0$, and $[S(SS)-NS(SS)]_{d=\frac{D}{2}} = 0$. Therefore,

$$S(SS) - NS(SS) \underset{>}{\overset{<}{-}} 0 \Leftrightarrow d \underset{>}{\overset{<}{-}} \frac{D}{2}. \tag{2}$$

This states that with spillovers, strategic incentives of R&D are larger than the non-strategic incentives for all $d \in (\frac{D}{2}, D]$.

Next we discuss the effect of spillovers on non-strategic and strategic incentives of R&D. We have:

$$NS(SS) - NS(WS) = \Pi(2D - d) - \Pi(2D) < 0 \ \ \forall d > 0. \tag{3}$$

This means, spillovers unambiguously reduce non-strategic R&D incentive.

Now,

$$S(SS) - S(WS) = \Pi(D + d) - \Pi(2d - D) - \Pi(D) + \Pi(-D)$$

Then it can be shown that,

$$9[S(SS) - S(WS)] = (6D - 2A - 3d)d$$

Therefore,

$$S(SS) - S(WS) > 0 \quad \text{for } 0 < d < \frac{2}{3}(3D - A) \equiv d^*. \tag{4}$$

Clearly, d^* is interior (i.e., $0 < d^* < D$) for $A < 3D < 2A$, and $d^* = D$ for $3D \geq 2A$.

From (3) and (4) we can write the following proposition.

Proposition 1 *While spillovers always reduce non-strategic incentive of R&D, it will increase strategic incentive if* $d \in (0, d^*)$ *and* $A < 3D$.

Now consider Nash equilibrium (NE) of each R&D game portrayed in Tables 1 and 2. We have the following results:

Proposition 2 (a) *When there are no spillovers,*
(i) (Y, Y) *is NE if* $S(WS) > 0$,
(ii) (Y, N) *and* (N, Y) *are Nash equilibria if* $S(WS) < 0 < NS(WS)$, *and,*
(iii) (N, N) *is the NE if* $NS(WS) < 0$.
(b) *When there are spillovers of R&D,*
(i) (Y, Y) *is NE if* $S(SS) > 0$,
(ii) (Y, N) *and* (N, Y) *are Nash equilibria if* $S(SS) < 0 < NS(SS)$, *and,*
(iii) (N, N) *is NE if* $NS(SS) < 0$.

The above conditions can be stated in terms of R&D investment explicitly. In general if R&D investment, R, is small, both firms will invest in R&D; if it is high, no firm will invest in R&D; and if it is in the intermediate range, only one firm will invest in R&D.

Now, we examine whether in equilibrium R&D incentives are larger in the presence of spillovers compared to no spillover case. In our scenario, spillovers will enhance R&D incentive if in equilibrium (i) no firm invests in R&D under no spillover case but at least one firm invests under spillovers, or (ii) only one firm invests under no spillover case, but both firms invest under spillovers.

First, suppose that (N, N) is the NE when there is no spillover of R&D and (Y, N) (or (N, Y)) is the NE when there are spillovers. This requires to satisfy the following two conditions simultaneously, that is,

(I) $NS(WS) < 0$ and $S(SS) < 0 < NS(SS)$.

But given that by (3), $NS(SS) < NS(WS)$, (I) cannot be satisfied. Hence, if (N, N) is the NE when there is no spillover, then (Y, N) or (N, Y) cannot be a NE when there are spillovers.

Now consider whether (N, N) is the NE when there is no spillover but (Y, Y) is the NE when there are spillovers. These two together require to satisfy the following inequality,

(II) $NS(WS) < 0 < S(SS)$.

We have,

$$NS(WS) - S(SS) = \Pi(2D) - \Pi(0) - \Pi(D + d) + \Pi(2d - D).$$

Then it can be shown that,

$$9[NS(WS) - S(SS)] = 4D(D - d) + 2d(A - D) + 3d^2.$$

Therefore, given $d \leq D < A$, we have,

$$NS(WS) - S(SS) > 0 \quad \forall d \in [0, D]. \tag{5}$$

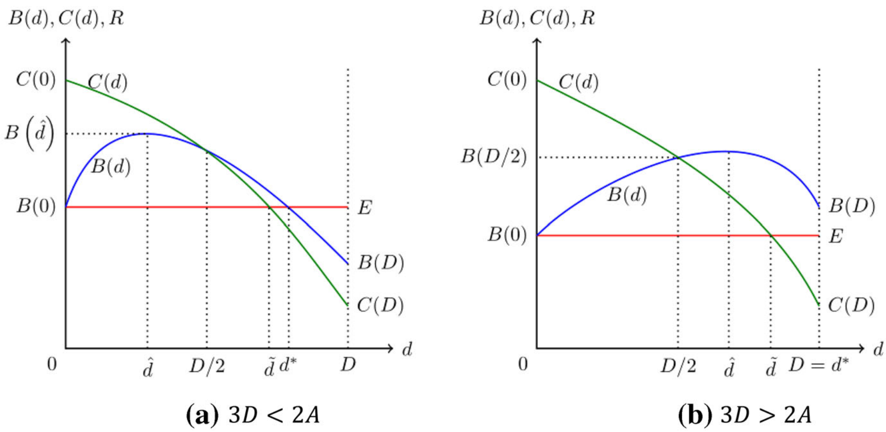

Fig. 1 **a** $3D < 2A$ and **b** $3D > 2A$

Hence (II) will never be satisfied. This means, (N, N) with no spillover and (Y, Y) with spillover cannot be simultaneously NE of the game.

Finally, we examine whether simultaneously (Y, N) and (N, Y) are Nash equilibria in the absence of spillover and (Y, Y) is the unique *NE* in the presence of spillovers. For this to hold we need to satisfy simultaneously the following three conditions:

$$\text{(IIIa)}\quad S(WS) < 0 < NS(WS)$$
$$\Leftrightarrow E \equiv \Pi(D) - \Pi(-D) < R < \Pi(2D) - \Pi(0) \equiv F$$
$$\text{(IIIb)}\quad S(SS) > 0 \Leftrightarrow B(d) \equiv \Pi(D + d) - \Pi(2d - D) > R, \text{ and}$$
$$\text{(IIIc)}\quad NS(SS) > 0 \Leftrightarrow C(d) \equiv \Pi(2D - d) - \Pi(0) > R.$$

Hence, all the above conditions will be satisfied if and only if,

$$E < R < \min\{F, B(d), C(d)\}.$$

Note that C(d) is strictly decreasing function in $[0, D]$ with maximum value C $(0) = \Pi(2D) - \Pi(0) = F$ and minimum value C(D) $= \Pi(D) - \Pi(0) < E$. On the other hand, $B(d)$ has a unique maximum in $[0, D]$ at $\widehat{d}$ where $\widehat{d} = \frac{3D-A}{3}$, and $B(\widehat{d}) < C(0)$. Further, $B(d)$ and C(d) have unique intersection at $d = \frac{D}{2}$.

From (4), we have $B(d) > E$. for all $d \in (0, d^*)$ where $d^* < D$ for $A < 3D < 2A$ and $d^* = D$ for $3D \geq 2A$. Then, $B(0) = E$ and $\widehat{d} \gtrless \frac{D}{2} \Leftrightarrow 3D \gtrless 2A$. Moreover, when $3D < 2A$, we have $C(D) < B(D) < E$ and when $3D > 2A$, we have $C(D) < E < B(D)$. Finally, let $C(d) = E$ occur at $d = \tilde{d}$. Then it follows that $\tilde{d} < d^*$. All the above relations are drawn in Fig. 1a, b.

Given the above, we can now write the main proposition of the paper.

Proposition 3 *Suppose innovation is non-drastic (i.e., $A > D$). Then for any given R&D investment $R \in (\underline{R}, \bar{R})$, $\underline{R} < \bar{R}$, there always exist two critical values of R&D spillover, $\underline{d}(R)$ and $\bar{d}(R)$, $\underline{d}(R) < \bar{d}(R)$, such that only one firm will invest in R&D*

under no spillover but both the firms will invest in R&D under spillover if and only if $d \in (\underline{d}(R), \bar{d}(R))$. *To be more precise, (i) if* $A < 3D < 2A$, *then for any* $R \in \left(B(0), B\left(\widehat{d}\right)\right)$, *we have* $\underline{d}(R) \in (0, \widehat{d})$ *and* $\bar{d}(R) \in (\widehat{d}, \tilde{d})$, *and (ii) if* $3D > 2A$, *then for any* $R \in \left(B(0), B\left(\frac{D}{2}\right)\right)$ *we have* $\underline{d}(R) \in (0, \frac{D}{2})$ *and* $\bar{d}(R) \in (\frac{D}{2}, \tilde{d})$.

Example 1 Suppose $a = 36, \quad c = 18, \quad D = 9$ and $R = 73$ Then we have: $\widehat{d} = 3$, $\frac{D}{2} = 4.5$, $d^* = 6$, $\tilde{d} = 4.823$, $\underline{d}(R) \approx 0.551$ and $\bar{d}(R) \approx 4.679$. Then our result holds for all $d \in (\underline{d}(R), \bar{d}(R))$. (For further details, see "Appendix 1".)

Our result clearly shows that even under complete information spillovers may enhance R&D incentives. This will certainly be the case when the R&D investment is neither too small nor too large and the spillover rate belongs to a specified interval defined by the size of R&D investment. Under this situation strategic incentives of firms are larger and each firm through R&D attempts to benefit from spillovers of other firm's knowledge.

Now it may be interesting to study the case when innovation remains non-drastic with spillovers but it becomes drastic in the absence of spillovers. This is equivalent to the assumption that $A \leq D < A + 2d$. Under this assumption, the payoff matrix under spillover as shown by Table 2 remains the same; however, in the payoff matrix of Table 1 we shall have $\Pi(\text{2D}) = \Pi^m(\text{D})$ and $\Pi(-\text{D}) = 0$, that is, when innovation is drastic without spillover, the firm which alone innovates becomes a monopolist, and the other firm ceases to operate.

With this change it is easy to see that under no spillover (Y, N) or (N, Y) will be the Nash equilibrium if and only if $\Pi(\text{D}) < R < \Pi^m(\text{D}) - \Pi(0)$. However, the conditions for (Y, Y) to be the unique Nash equilibrium under spillover remain the same as before.

Therefore, with drastic innovation under no spillover case, the critical lower limit of R goes up for spillover to increase R&D incentives (because $\Pi(-\text{D}) = 0$ in the expression of E). This means, the critical interval of R gets reduced compared to non-drastic case. This also means the critical interval of d is shortened (because $\underline{d}(R)$ becomes higher and $\bar{d}(R)$ becomes lower).

Thus when innovation is drastic under no spillover but non-drastic with spillover, the possibility that spillovers will increase R&D incentives of firms will be reduced compared to the case of non-drastic innovation with and without spillover. The following example shows that the innovation is drastic under no spillover but non-drastic with spillover. Then spillover increases R&D incentives.

Example 2 Let $a = 12$, $c = 9$, $D = 6$, $d = 3$ and $R = 10$. Then, $A < D < A + 2d$, $\Pi(-\text{D}) = 0$, $\Pi(0) = 1$, $\Pi(\text{D}) = 9$, $\Pi^m(D) = 20.25$, $\Pi(\text{D+d}) = 16$, $\Pi(\text{2d} - \text{D}) = 1$, $\Pi(\text{2D} - \text{d}) = 16$. Given the payoffs, (Y, N) (or, (N, Y)) is a NE under no spillover but (Y, Y) is the unique NE under spillover.

One relevant question in the context could be whether our basic result would go through under price competition. In "Appendix 2" we have sketched the model of differentiated price competition and have shown that there are situations where spillover increases R&D incentive of a competing firm. This proves that our result is not specific to the assumption of quantity competition only, but rather it is robust with respect to product market competition.

Conclusion

It is very common to believe that in a complete information structure the presence of spillovers in R&D will reduce R&D investment of a firm. Presence of spillovers implies that some R&D knowledge of an innovator leaks out to its contenders; hence the innovator cannot get the full benefit of its R&D investment. This induces the firm to under-invest in R&D. On the other hand, there are some empirical literature that show the possibility of enhanced R&D under spillovers. Theoretically this is explained in the context of incomplete information. In the present paper we have shown that enhanced R&D under spillovers may hold even under complete information. We argue that when all the competing firms invest, each firm gets the benefit from other firms' investment and the total *ex ante* gain can be large enough to induce the firm to invest in R&D. Actually, in such a situation spillovers act as network externalities.

We have presented a duopoly model whence the firms interact in R&D and production, and R&D investment is characterised by spillover of R&D knowledge. We have shown that under some conditions both firms will invest in R&D when there are spillovers, although not all of the firms would invest in R&D in the absence of spillovers of knowledge. This actually happens when the size of R&D investment is neither too large nor too small and the spillover rate belongs to an interval defined by the R&D investment. Under these conditions only one firm invests in equilibrium when there are no spillovers but both the firms invest in the presence of spillovers. However, if in equilibrium no firm invests in research when there is no spillover, we find that no firm will have any incentive to invest in research when there are spillovers.

Finally, to mention, our result does not depend on the specific nature of product market competition. We have considered competition in both quantity and price and have shown that spillovers may enhance R&D incentives of the interacting firms. Hence our result is quite general with respect to product market competition.

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Appendix 1

Given the numerical values of the parameters in Example, further assume that $d = 3$. The we have: $A = 18$, $\Pi(-\mathrm{D}) = 9$, $\Pi(0) = 36$, $\Pi(\mathrm{D}) = 81$, $\Pi(2\mathrm{D}) = 144$, $\Pi(\mathrm{D}+\mathrm{d}) = 100$, $\Pi(2\mathrm{d}-\mathrm{D}) = 25$, $\Pi(2\mathrm{D}-\mathrm{d}) = 121$

Given above, the payoff matrix when there is no spillover is:

		Firm 2	
		Y	N
Firm 1	Y	8, 8	71, 9
	N	9, 71	36, 36

For this payoff matrix, (Y, N) and (N, Y) are two Nash equilibria. The payoff matrix when there is spillover is given below:

		Firm 2	
		Y	N
Firm 1	Y	27, 27	48, 25
	N	25, 48	36, 36

Clearly in this game (Y, Y) is the unique Nash equilibrium.

Appendix 2

Let the demand function as faced by firm i be given by $q_i = a - p_i + \gamma p_j$ where $i = 1, 2 (i \neq j)$ and $\gamma \in (0, 1)$. For the marginal costs c_1 and c_2 of firm 1 and firm 2 respectively, the profit function of firm i is $\pi_i = (p_i - c_i)q_i$. Then under price competition, the equilibrium prices and quantities are respectively, $p_i = \frac{(2+\gamma)a+2c_i+\gamma c_j}{4-\gamma^2}$ and $q_i = \frac{(2+\gamma)a-(2-\gamma^2)c_i+\gamma c_j}{4-\gamma^2}$, therefore, $\pi_i = (q_i)^2$.

Define $K = (2+\gamma)a - (2-\gamma-\gamma^2)c$ and $\pi(x) = \left(\frac{K+x}{4-\gamma^2}\right)^2$. Then the payoff matrix under no spillover is given by:

		Firm 2	
		Y	N
Firm 1	Y	$\pi\big((2-\gamma-\gamma^2)D\big) - R$, $\pi((2-\gamma-\gamma^2)D) - R$	$\pi((2-\gamma^2)D) - R$, $\pi(-\gamma D)$
	N	$\pi(-\gamma D)$, $\pi((2-\gamma^2)D) - R$	$\pi(0)$, $\pi(0)$

We have $S(WS) = \pi\big((2-\gamma-\gamma^2)D\big) - \pi(-\gamma D) - R$ and $NS(WS) = \pi\big((2-\gamma^2)D\big) - \pi(0) - R$, then $NS(WS) > S(WS)$.

The payoff matrix under spillover is given by:

Firm 2

		Y	N
Firm 1	Y	$\pi((2-\gamma-\gamma^2)(D+d))-R$, $\pi((2-\gamma-\gamma^2)(D+d))-R$	$\pi\big((2-\gamma^2)D-\gamma d\big)-R$, $\pi\big((2-\gamma^2)d-\gamma D\big)$
	N	$\pi\big((2-\gamma^2)d-\gamma D\big)$, $\pi\big((2-\gamma^2)D-\gamma d\big)-R$	$\pi(0)$, $\pi(0)$

Here, $S(SS) = \pi\big((2-\gamma-\gamma^2)(D+d)\big) - \pi\big((2-\gamma^2)d-\gamma D\big) - R$, and $NS(SS) = \pi\big((2-\gamma^2)D-\gamma d\big) - \pi(0) - R$. Then, $S(SS) \lesseqgtr NS(SS) \Leftrightarrow d \lesseqgtr \frac{\gamma}{2-\gamma^2}D$. Further, at $d = 0$, we have $NS(SS) > S(SS)$, and at $d = D$ we have $NS(SS) < S(SS)$. Finally, $S(SS)$ is maximized at $\widehat{d} = \frac{\left[(2-\gamma-\gamma^2)^2-\gamma(2-\gamma^2)\right]D-\gamma K}{(2-\gamma^2)^2-(2-\gamma-\gamma^2)^2}$ iff $\left[(2-\gamma-\gamma^2)^2 - \gamma(2-\gamma^2)\right]D > \gamma K$, otherwise, $\widehat{d} = 0$.

Following our earlier notation, $E = S(WS)+R$, $B(d) = S(SS)+R$ and $C(d) = NS(SS) + R$. Therefore, $B(0) = E$ and $C(D) < E$.

Finally, if $\left[(2-\gamma-\gamma^2)^2 - \gamma(2-\gamma^2)\right]D > \gamma K$ holds, then we have a range of R and correspondingly an interval of d for which (Y, N) or (N, Y) is a NE under no spillover but (Y, Y) is the unique NE under spillover. The relevant condition is necessarily satisfied, for example, for $D = 2$, $K = 4$ and $\gamma = 0.25$.

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